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Clustering of Faint Galaxies: $\omega(\theta)$, Induced by Weak Gravitational Lensing

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ABSTRACT
Weak gravitational lensing by large scale structure affects the number counts of faint galaxies through the "magnification bias" and thus affects the measurement of the angular two-point correlation function $\omega(\theta)$. At faint magnitudes the clustering amplitude will decrease differently with limiting magnitude than expected from Limber's equation. The amplitude will hit a minimum and then rise with limiting magnitude. This behavior occurs because $\omega(\theta)$ due to clustering decreases with distance, while the "magnification bias" due to weak lensing increases with distance. The apparent magnitude m_{min} at which the magnification bias starts to dominate the observed clustering is model and color dependent. It is given by $\omega(m = m_{min}, \theta = 5') \approx (1 - 2) \times 10^{-3}(5s-2)^2\Omega_0^2\sigma_8^2$, where s is the logarithmic slope of the number counts. Already published measurements of $\omega(\theta)$ at $R = 25$ may be strongly influenced by the "magnification bias". An experiment using the ratio of blue and red number counts across the sky can be designed such that the effects of the "true" clustering is minimized. The magnification bias is a measurement of the clustering of the mass. This weak lensing experiment does not require measuring shapes and position angles of galaxies. I derive a revised Limber's Equation including the effects of magnification bias.

Key words: galaxies: clustering - cosmology: observations - gravitational lensing - large scale structure of the Universe

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1 INTRODUCTION

One of the most useful statistics for the evolution and clustering properties of faint, distant galaxies is the two-point angular correlation function $\omega(\theta)$. It measures the excess number of pairs of galaxies separated by angle θ on the sky. For a given galaxy sample, $\omega(\theta)$ depends on the redshift distribution of the galaxies, and the three dimensional clustering amplitude $\xi(r, z)$. As pointed out by Koo & Szalay (1984), $\omega(\theta)$ can be used to constrain models of the evolution and clustering of faint galaxies at intermediate and high redshift. A large number of investigations have measured the correlation for faint magnitudes in different passbands, *eg.* Bernstein *et al.* 1994; Brainerd, Smail, & Mould 1995 (BSM); Couch, Jurcevic, & Boyle 1993; Efstathiou *et al.* 1991; Neuschaefer, Windhorst, & Dressler 1991; Roche *et al.* 1993. There is a general consensus that $\omega(\theta)$ can be approximated by a power law with slope of -0.8 and an amplitude that decreases with increasing limiting magnitude. There is, however, little consensus on what that means in terms of luminosity evolution, merger history, and clustering. The number counts as a function of magnitude, (*eg.* Lilly *et al.* 1991, Smail *et al.* 1995), and the redshift surveys to faint magnitudes (*eg.* Broadhurst *et al.* 1992, Glazebrook *et al.* 1994) put significant constraints on the galaxy evolution.

Weak gravitational lensing of distant galaxies, equivalent to a systematic induced magnification and distortion without multiple imaging, is a probe of the large scale mass distribution of the universe, (*eg.* Gunn 1967). Essentially, when a light ray from a distant galaxy passes through an overdense region in mass, the image will be magnified and distorted tangentially with respect to the center of the mass overdensity. This effect has been measured in a number of galaxy clusters, [*eg.* Tyson,

Valdes, & Wenk (1990), Bonnet *et al.* (1994)]. Conversely, passing through a low density region, the image will become dimmer and elongated radially. Independent perturbations along the line of sight add up stochastically [Blandford *et al.* 1991(BSBV), Miralda-Escudé 1991, Kaiser 1992, Villumsen 1995a(V95a)]. Weak lensing by large scale structure is observable because galaxy images near each other on the sky will be sheared coherently leading to a locally preferred direction. Due to the weak nature of the lensing there has been no undisputed measure of the distortion, see f.ex. (Mould *et al.* 1994; Kaiser, Squires, & Broadhurst 1995), though a reanalysis by Villumsen (1995b) shows a tentative detection at the 5-σ level consistent with expectations from a weakly biased flat Cold Dark Matter (CDM) universe. The problem with weak lensing is that it is weak. Measuring the weak shear requires measuring the shape and orientation of distant galaxy images. This is observationally feasible, but quite difficult, since the images are small and faint, and the possible systematic errors are the limiting factors.

Weak gravitational lensing is characterized by the convergence κ and shear γ. We will work only in the weak limit, *i.e.* $|\kappa|, |\gamma| \ll 1$. In this limit, both κ and γ are observables. The distortion of an image is given by γ, such that an intrinsically round image will aquire an eccentricity equal to $|\gamma|$ and the phase of γ will be twice the position angle of the major axis. The magnification A of an image is $A = 1 + 2\kappa$. In clusters, the distortion is used to generate shear maps which can then be converted into maps of the surface density as pioneered by Kaiser & Squires (1993) and further developed by Seitz & Schneider (1995). Recently Broadhurst, Taylor & Peacock(1995) have demonstrated the use of the magnification to improve the information about the surface mass density.

In this paper I propose a new measure of weak gravitational lensing by large scale structure based on the magnification bias introduced by Turner (1980) and used by Broadhurst (1995) for lensing in clusters. Basically, weak lensing will induce angular correlations in the number density of galaxies on the sky. This measure only involves the convergence κ and does not involve γ. Thus, it is not necessary to measure the shape of high-redshift galaxies. This means that a galaxy survey to look for magnification bias can be pushed to fainter magnitudes than a search for galaxy distortions.

In §2 I estimate the amplitudes of "true" and "apparent" clustering, in §3 I work out the theory. In §4 I calculate $\omega(\theta)$ in terms of the 3-D correlation function and in §5 I discuss the implications.

2 ESTIMATES OF "TRUE" AND "APPARENT" CLUSTERING

Magnification bias is the change in surface number density of sources on the sky due to gravitational lensing (*eg.* Turner *et al.* 1984). As the total number of galaxies on the sky is conserved, magnification in a given patch of the sky will lead to a decrease in the number density of galaxies, because you are looking at a smaller area of sky than you think. However, magnification will also brighten an image so more galaxies will be visible at a given apparent magnitude m. If the number counts $N_0(m)$ in the absence of lensing has a slope

$$s = \frac{d\ \log N_0(m)}{dm}, \tag{1}$$

then the number counts will be changed

$$N_{obs}(m) = N_0(m) A^{2.5s-1} = N_0(m)(1 + (5s-2)\kappa), \tag{2}$$

where $N_{obs}(m)$ and $N_0(m)$ are the number counts in the presence/absence of lensing. The first equality is generally valid, the second assumes that $|\gamma|, |\kappa| \ll 1$. For $s = 0.4$ there is no magnification bias, for $s < 0.4$ there is a depletion of counts, while for $s > 0.4$ there is an increase in counts in the presence of magnification.

In order to estimate the clustering effects of the "magnification bias", assume a universe with no intrinsic clustering of galaxies. Then weak lensing will introduce an apparent clustering characterized by an angular two-point correlation function $\omega_{\kappa\kappa}(\theta)$

$$\omega_{\kappa\kappa}(\theta) \equiv \frac{\left\langle \left[N_{obs}(\bar{\phi}+\bar{\theta}, m) - N_0(m)\right] \left[N_{obs}(\bar{\phi}, m) - N_0(m)\right] \right\rangle}{N_0(m)^2}. \tag{3}$$

Here the correlation function is calculated as the average over all direction vectors $\bar{\phi}$ of the relative excess number of galaxy pairs separated by angle $|\bar{\theta}| = \theta$. We obtain from Eqs. (2,3)

$$\omega_{\kappa\kappa}(\theta) = \frac{\left\langle \left[N_0(m)(5s-2)\kappa(\bar{\phi}+\bar{\theta})\right] \left[N_0(m)(5s-2)\kappa(\bar{\phi})\right] \right\rangle}{N_0(m)^2} \tag{4}$$

$$= (5s-2)^2 \left\langle \kappa(\bar{\phi}+\bar{\theta})\kappa(\bar{\phi}) \right\rangle \equiv (5s-2)^2 C_{\kappa\kappa}(\theta) \ = \ (5s-2)^2 C_{pp}(\theta). \tag{5}$$

Here, $C_{\kappa\kappa}(\theta)$, and $C_{pp}(\theta)$ are the correlation functions of the convergence and the shear. In this weak limit they are identical (BSBV and V95a). The shear and convergence correlation functions are steeply increasing functions of the comoving angular distance y, and thus of redshift z. For a universe with $\Omega_0 = 1,\ \Omega_\Lambda = 0$,

$$\omega_{\kappa\kappa} \propto C_{\kappa\kappa} \propto y^3 = 8\left(1 - (1+z)^{-1/2}\right)^3 . \tag{6}$$

The variation in the number counts to a given magnitude limit across the sky is an observable. The shear and convergence correlation functions are related to the density distribution and cosmological parameters as described in BSBV, V95a. The shear γ will also influence the number counts, but the effect is quadratic in γ and can be ignored.

In real life, of course, we observe a true clustering of galaxies at faint magnitudes characterized by the angular two-point correlation function $\omega_{\mu\mu}(\theta)$, (see references in §1). The correlation induced by magnification bias $\omega_{\kappa\kappa}(\theta)$ is of practical interest only if it becomes significant compared to $\omega_{\mu\mu}(\theta)$. In a universe where the clustering is constant in comoving coordinates, $\omega_{\mu\mu}(\theta)$ will fall off roughly inversely with distance. Lensing is a cumulative effect while the intrinsic three dimensional clustering will be diluted by projection effects. This would indicate that at low redshift $\omega_{\kappa\kappa}(\theta)$ is unimportant but that at sufficiently high redshift $\omega_{\kappa\kappa}(\theta)$ might become important. The observational issue is whether this occurs at observable apparent magnitudes.

The current status of $\omega(\theta)$ for red samples is summarized by (BSM), see Fig. 2. It is common to characterize the correlation function as a power law with slope γ and the amplitude at 1 degree, A_w. Then $\omega(\theta) \approx A_w\theta^{-\gamma}$, θ measured in degrees. There is a general consensus that $\gamma \approx 0.8$ and that A_w is a decreasing function of depth. (BSM) state that the amplitude from red counts is $A_w \propto R^{-0.27\pm0.01}$ with $A_w(R = 25) \approx 3 \times 10^{-4}$.

For a CDM universe where the characteristic redshift of the sources is $z \sim 1$, the amplitude of the polarization correlation is given by the cosmological density parameter Ω_0 and the rms density fluctuations on a scale of 800 km s^{-1}, σ_8 (BSBV, V95a),

$$C_{\kappa\kappa}(\theta = 5') \approx 1.5 \times 10^{-3}\Omega_0^2\sigma_8^2. \tag{7}$$

This would indicate that the amplitude of A_w induced by lensing is

$$A_{w,\kappa} \approx 2 \times 10^{-4}(5s - 2)^2\Omega_0^2\sigma_8^2. \tag{8}$$

The Mould *et al.* (1994), presented in (BSM) were obtained in the r band where the source counts have $s_r = 0.3$. Then

$$A_{w,\kappa}(r) \approx 5 \times 10^{-5}\Omega_0^2\sigma_8^2. \tag{9}$$

This is lower than the observed value of the correlation function $A_w(R = 25) \approx 3\ 10^{-4}$, $A_w(R = 25.5) \approx 1.5\ 10^{-4}$, but not negligible. However, we have neglected the correlation between the magnification bias and the true clustering. If light traces mass, then there will be a strong correlation between the magnification and the true clustering. If $s < 0.4$, this is really an anticorrelation and the observed clustering will be less than the true clustering.

Smail *et al.* (1995) find that the slope in the VRI bands all tend to $s \approx 0.3$ for the faintest galaxies. Neuschaefer *et al.* (1991) have reported on $\omega(\theta)$ in the Gunn-g band and they find a slope $s_g \approx 0.45$. This is close to the critical slope of $s = 0.4$, so the magnification bias is expected to be small and quite uncertain. For $g < 24.5$ they find that $\omega(\theta)$ is approximately a power law with the amplitude decreasing with increasing magnitude. For fainter magnitudes they find a very steep rise in the amplitude. This rise is much too steep to be accounted for by magnification bias. At $g \approx 24.5$, $\omega(\theta = 1') \approx 10^{-2}$. For a high value of $\Omega_0\ \sigma_8$ there may be a small contribution to $\omega(\theta)$ from magnification bias, though this is uncertain.

However, Broadhurst (1995) has shown that for the I band counts, $I > 24$, the red counts, $V - I > 2.0$, are fitted well by a slope $s_r \approx 0.15$. The blue counts, $V - I < 1.0$, have $s_b \approx 0.5$. The prediction is thus that for the blue counts, there will be a measurable magnification bias. For the red counts, though, the prediction is that the magnification bias will be strong,

$$A_{w,\kappa}(r) \approx 3 \times 10^{-4}\Omega_0^2\sigma_8^2. \tag{10}$$

This is comparable to the observed correlation function amplitude.

These results indicate that at accessible apparent magnitudes the magnification bias will give a significant contribution to the observed angular two-point correlation function. This encourages a more detailed study of the magnification bias.

3 THEORY

3.1 Single Sample Statistics

In the limit where the intrinsic clustering and the magnification bias are weak, the clustering, characterized by the angular two-point correlation function $\omega(\theta)$ can be calculated. The magnification bias works on the "true" number counts, *i.e.* the number counts which include the true clustering,

$$N_{obs}(m) = N_0(m)(1 + \Delta\mu)A^{2.5s-1} \approx N_0(m)(1 + \Delta\mu + (5s - 2)\kappa). \tag{11}$$

Here, $\Delta\mu$ is the true fractional excess number of sources in a particular patch on the sky. From this we can calculate $\omega(\theta)$ from an average over the sky,

$$\omega(\theta) \quad = \quad \frac{\left\langle \left[N_{obs}(\bar{\phi} + \bar{\theta}, m) - N_0(m)\right] \left[N_{obs}(\bar{\phi}, m) - N_0(m)\right] \right\rangle}{N_0(m)^2} \tag{12}$$

$$
\begin{aligned}
&= \left\langle \left[\Delta\mu(\bar\phi+\bar\theta) + (5s-2)\kappa(\bar\phi+\bar\theta)\right]\left[\Delta\mu(\bar\phi) + (5s-2)\kappa(\bar\phi)\right]\right\rangle && (13)\\
&= \left\langle \Delta\mu(\bar\phi+\bar\theta)\Delta\mu(\bar\phi)\right\rangle + (10s-4)\left\langle \Delta\mu(\bar\phi+\bar\theta)\kappa(\bar\phi)\right\rangle + (5s-2)^2\left\langle \kappa(\bar\phi+\bar\theta)\kappa(\bar\phi)\right\rangle && (14)\\
&\equiv \omega_{\mu\mu}(\theta) + 2\omega_{\mu\kappa}(\theta) + \omega_{\kappa\kappa}(\theta) && (15)\\
&= \omega_{\mu\mu}(\theta) + 2\omega_{\mu\kappa}(\theta) + (5s-2)^2 C_{\kappa\kappa}(\theta). && (16)
\end{aligned}
$$

Again, the average is over all direction vectors $\bar\phi$. The three individual terms are not observables, only the sum. The first term is the true galaxy clustering, the third term is the apparent clustering induced by the magnification bias, and the second term is a cross correlation. The magnification bias is induced by mass density fluctuation, while the true galaxy clustering measures the galaxy number density fluctuations. If in projection, the number density fluctuations are decoupled from the mass density fluctuation, then the cross term will be zero.

The observed number counts of galaxies as a function of magnitude in a given passband and position on the sky will depend in a complicated, and unknown way, on the formation and evolution of galaxies. In the calculations we are going to use the comoving radial distance x as fundamental variable,

$$
\begin{aligned}
x(z) &= \int_0^z dz'\; H^{-1}\left(z'\right)\;;\; H(z') = \left[\Omega_0(1+z')^3 + (1-\Omega_0-\Omega_\Lambda)(1+z')^2 + \Omega_\Lambda\right]^{1/2}, && (17)\\
y(z) &= \frac{\sinh\left[\sqrt{(1-\Omega_0-\Omega_\Lambda)}\; x(z)\right]}{\sqrt{(1-\Omega_0-\Omega_\Lambda)}}. && (18)
\end{aligned}
$$

In a simple model, characterised by a selection function $S(x)$, and linear bias factor $b(x)$ we can calculate statistically the surface density distribution. We define $S(x)$ to be the mean comoving number density of observed objects at distance x. The function S can also be thought of as a function of redshift, or lookback time. The normalisation of S is such that

$$\int_0^\infty dx\; y^2 S(x) = 1. \tag{19}$$

The function S has hidden in it the number density evolution, the luminosity evolution, and the differential K-correction. We define the linear bias factor b as

$$b(x) = \left(\frac{\delta N}{N}\right) \Big/ \left(\frac{\delta\rho}{\rho}\right), \tag{20}$$

so b is not a function of scale but can be a function of epoch. This linear biasing scheme assumes that the galaxies cluster the same way as the mass, just at a possibly different amplitude characterized by b.

The surface density fluctuation $\Delta\mu$ can then be written as a line integral in comoving radial distance x of the fractional volume overdensity $b(x)\delta(\underline{x})$ times the volume element $dx \cdot y^2$ with a selection function $S(x)$. Here y is the comoving angular diameter distance.

$$
\begin{aligned}
\Delta\mu(\bar\theta) &= \int_0^\infty dx\; y^2 S(x) b(x) \delta(\underline{x}), && (21)\\
\kappa(\bar\theta) &= 3\Omega_0 \int_0^\infty dx\; y w(x) \delta(\underline{x})/a. && (22)
\end{aligned}
$$

In principle, all the line integrals are only out the horizon distance x_H, however, we can just set $S(x)$ equal to zero for $x > x_H$. The equation for κ is taken from V95a, Eq 29, where $\kappa \equiv -\Delta M$. Here, $\delta(\underline{x})$ is the density perturbation at position $\underline{x}$ evaluated at the epoch when the lightray passes through. The expansion factor a also enters, $a = 1$ at the present epoch. The function $w(x)$ is the lensing selection function which for a lens at distance x is the integral over all sources $y^2 S(x')$ further away than x of the ratio of the lens-source distance y_{LS} and the observer-source distance y_{OS}.

$$w(x') = \int_{x'}^\infty dx\; y^2 S(x) \frac{y_{LS}}{y_{OS}} \tag{23}$$

For convenience we write $\delta(\underline{x}) \equiv f(x) a \delta_0(\underline{x})$, where $\delta_0(\underline{x})$ is the density fluctuation evaluated today. We implicitly assume that the "growth factor" $f(x)$ is a universal function of expansion factor $a(x)$ only. Then

$$\Delta\mu(\bar\phi) + (5s-2)\kappa(\bar\phi) = \int_0^\infty dx\; y\; f(x)\left[y\; S(x) a\; b(x) + 3\Omega_0(5s-2) w(x)\right]\delta_0(\underline{x}). \tag{24}$$

Suppose we have sufficiently well behaved functions $F(\bar\phi)$, and $G(x)$ defined as

$$F(\bar\phi) \equiv \int_0^{X_H} dx G(x) \delta_0(\underline{x}). \tag{25}$$

The two-point correlation function $C_{FF}(\theta)$ of F will then be

$$C_{FF}(\theta) = 4\pi^2 \int_0^\infty dx\ G^2(x) \int_0^\infty dk\ k\ P(k) J_0(kx\theta). \tag{26}$$

Equation 24 is of this form so we can write the two-point correlation function as

$$\omega(\theta) \quad = \quad 4\pi^2 \int_0^\infty dx\ y^2\ f^2(x) \left[yS(x)a\ b(x) + 3\Omega_0(5s-2)w(x)\right]^2 \int_0^\infty dkkP(k)J_0(kx\theta). \tag{27}$$

We see in Eq.(27) that if $s > 0.4$, the weak lensing will give a positive contribution to the correlation amplitude, while if $s < 0.4$, the weak lensing will have a negative contribution to the correlation amplitude.

The weak lensing can change the angular dependence of $\omega(\theta)$ if $P(k)$ is not a power law. On a given angular scale θ, $\omega(\theta)$ samples decreasing wavenumbers as a function of x as seen in Eq.27. The weighting in k is given by the outer integral which is changed by the magnification bias. Thus, if $P(k)$ is not a power law $\omega(\theta)$ will be changed by the magnification bias.

At low redshift the intrinsic clustering will dominate and the magnification bias is negligible. At higher redshift, the intrinsic clustering will drop and the magnification bias will increase dramatically. In the case of flat number counts, *i.e.* $s < 0.4$, the angular two-point correlation function will decrease faster with depth than expected from Limber's equation (Peebles 1980). Eventually the contibutions from the true clustering and the magnification will be about equal and the observed clustering amplitude will hit a minimum. At even higher redshift the magnification bias will dominate and $\omega(\theta)$ will increase.

For steep number counts, *i.e.* $s > 0.4$, the correlation amplitude will fall slower with apparent magnitude than expected from Limber's equation, reach a minimum, and then rise again. In both cases the correlation amplitude will eventually become an increasing function of depth. The magnitude at which this happens will depend on cosmological parameters, Ω_0 and b.

How do the intrinsic correlations and the apparent correlations scale with depth of the sample. Take a simple example where $\Omega_0 = 1$, $\Omega_\Lambda = 0$, $1 - a \ll 1$, and $b(x)$ is a constant, then $y = x$. In Eq.(27) in the outer integral, the contribution from intrinsic clustering will scale inversely with q, while the contribution from magnification bias will scale as q^3. In Eq. 27 the calculation of the intrinsic clustering involves five powers of the distance. As S^2 is inversely proportional to the sixth power of the characteristic distance of the sources we have that $\omega_{\mu\mu}(\theta)$ scales inversely with characteristic distance of the sources. Also in Eq. 27 we see that $\omega_{\kappa\kappa}(\theta)$ scales as the third power of the characteristic distance. The inner integral in k is the same for the two terms, and is proportional to x^{-2-n} for a power law power spectrum with slope n.

We can thus estimate the sample distance at which the magnification bias will dominate. The correlations induced by weak lensing are approximately $\omega_{\kappa\kappa}(\theta) \propto \Omega_0^2\sigma_8^2 y^{1-n}$ while the true angular correlation has $\omega_{\mu\mu}(\theta) \propto \sigma_8^2 b^2 y^{-3-n}$. Thus the characteristic distance y_e at which the two effects become comparable is $y_e^{-2} \propto \Omega_0/b$. The limiting magnitude m_{min} at which the correlation amplitude is at a minimum is an observable. If the typical sample redshift is $z \approx 1$ then $m_{\rm min}$ is determined from the equation.

$$\omega(m = m_{\rm min}, \theta = 5') \approx (1\ -\ 2) \times 10^{-3}(5s-2)^2\Omega_0^2\sigma_8^2. \tag{28}$$

In Figure 1 we demonstrate the effects of the magnification bias through a simple model. Assume that $S(x)$ is constant out to some distance x_0. Assume that $b = 1/\sigma_8$, *i.e.* the bias is independent of epoch. Further assume $\Omega_0 = 1$ and $P(k) \propto k^{-1.2}$. In this model, which should be seen as an illustration only, we can calculate A_W as a function of mean source redshift $< z >= \int_0^\infty dx\ z(x)x^2S(x)$, for various values of s and b. In all these models the intrinsic clustering is the same.

The solid curve, $s = 0.4$, is due entirely to the intrinsic clustering, there is no magnification bias. The uppermost curve, $s = 0.5,\ b = 1$, shows a positive magnification bias and falls off slower than the true clustering. At $< z >\approx 1$ the observed clustering amplitude is 70% higher than the true amplitude. At higher redshifts A_w is quite flat and at $< z >\approx 2.5$ it will begin to rise.

The two curves for $s = 0.3$, which are appropriate for red samples, originally fall faster than the true correlation. For $z \lesssim 1.8$, the less biased model, dotted line, will have a lower amplitude of A_w than the more biased model, while at higher redshifts the opposite is true. This behavior can be understood as follows. The magnification bias consists of two terms, $\omega_{\kappa\kappa}(\theta)$ which is always a positive contribution and quadratic in σ_8, and $\omega_{\mu\kappa}(\theta)$ which is linear in σ_8 and is negative. At low redshift the negative linear term dominates while at sufficiently high redshift the positive quadratic term dominates.

For an unbiased population of galaxies with $s = 0.2$, the magnification bias is so strong that the correlation amplitude hits a minimum at $< z >\approx 1.3$ and is larger than the true amplitude for $< z > \gtrsim 1.5$. The curve for $b = 2$, $s = 0.2$ is identical to the curve for $b = 1$, $s = 0.3$.

If we observe the correlation function $\omega(\theta)$ for a single limiting magnitude we cannot tell what is the intrinsic contribution and what is magnification bias, it is necessary to use a range of limiting magnitudes. The slope of the number counts s is an observable, so if $s < 0.4$, and the correlation amplitude is decreasing with limiting magnitude, we know that the intrinsic correlation is higher than the observed correlation. If the amplitude is rising we cannot make this inference. If $s > 0.4$ we know that the true correlation is less than the observed correlation.

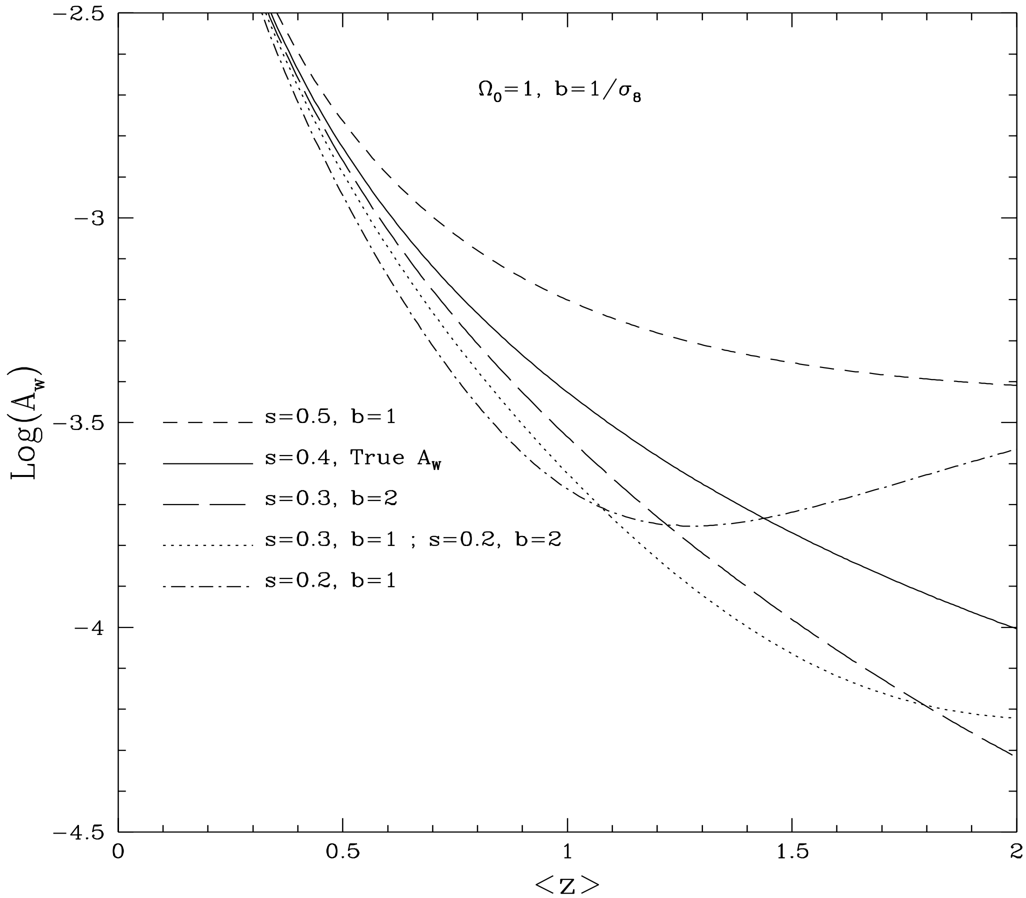

Figure 1. Correlation Amplitude A_w as a function of mean redshift and slope of number counts. The solid curve is the intrinsic correlation. The dotted, long dashes, dash-dotted curves are relevant for red galaxy samples. The short-dash curve is relevant for blue galaxy samples.

If an increase in correlation amplitude as a function of limiting magnitude is observed, we see that it is not necessary to invoke a local population of weakly clustered intrinsically faint galaxies. It is also not necessary to assume that we are seeing a strongly clustered population of distant galaxies.

These models should be seen as an illustration only. In order to properly model the correlation function we need to consider luminosity evolution, merger history, density evolution, differential K-correction etc. In particular we need to relate $< z >$ to limiting magnitude.

From Figure 1, we see that if the data at $R \approx 25.0 - 25.5$ in (BSM) is at $< z > \approx 1$, there is likely to be a significant contribution from magnification bias in the data.

3.2 Two-Sample Statistics

An equivalent statistic involving two samples can be used. The number counts of sources selected in two separate ways in the sample field are used. The ratio of counts in two passbands at magnitudes (m_1, m_2) can be calculated in the case of weak intrinsic clustering of galaxies.

$$\Delta(m_1, m_2, \bar{\phi}) \equiv \frac{N_{obs}(m_1, \bar{\phi})}{N_0(m_1)} \Bigg/ \frac{N_{obs}(m_2, \bar{\phi})}{N_0(m_2)} - 1 \approx \frac{1 + \Delta\mu(m_1, \bar{\phi}) + [5s_1 - 2]\kappa_1(\bar{\phi})}{1 + \Delta\mu(m_2, \bar{\phi}) + [5s_2 - 2]\kappa_2(\bar{\phi})} - 1 \quad (29)$$

$$\approx \Delta\mu(m_1, \bar{\phi}) - \Delta\mu(m_2, \bar{\phi}) + 5\left[s_1\kappa_1(\bar{\phi}) - s_2\kappa_2(\bar{\phi})\right] - 2\left[\kappa_1(\bar{\phi}) - \kappa_2(\bar{\phi})\right]. \quad (30)$$

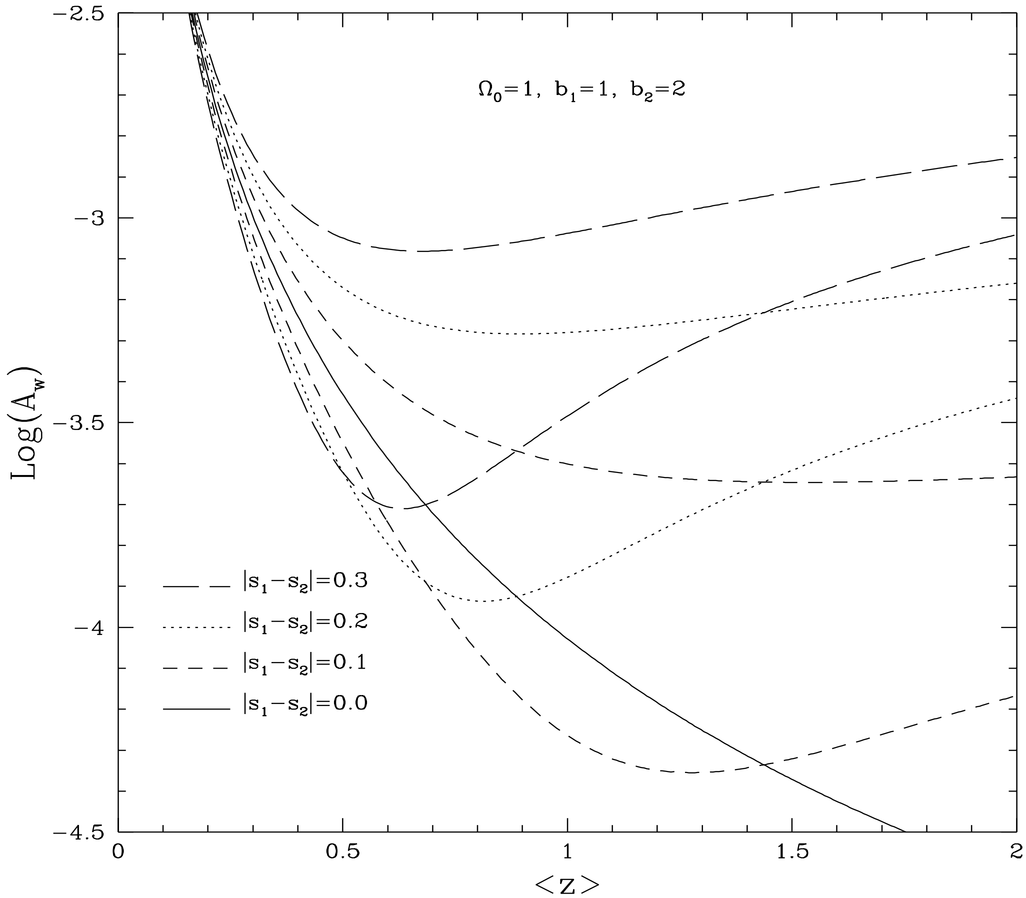

Figure 2. Correlation Amplitude A_w as a function of mean redshift and difference in slope of number counts. The three curves that start below the solid curve are for $s_1 < s_2$. The three other curves are for $s_1 > s_2$.

(m_1, m_2), (s_1, s_2), (κ_1, κ_2) are the magnitudes, slopes of the number counts and the convergences in the two passbands. The convergences need not be identical since the depth of the samples in redshift need not be the same in the two passbands. Equivalently, we can look at two separate samples selected by color, in a single passband. This statistic in the weak clustering regime is equivalent to the statistic involving the difference in number counts.

$$\Delta(m_1, m_2, \bar{\phi}) \equiv \frac{N_{obs}(m_1, \bar{\phi})}{N_0(m_1)} - \frac{N_{obs}(m_2, \bar{\phi})}{N_0(m_2)} \quad (31)$$

$$= \Delta\mu(m_1, \bar{\phi}) - \Delta\mu(m_2, \bar{\phi}) + 5\left[s_1\kappa_1(\bar{\phi}) - s_2\kappa_2(\bar{\phi})\right] - 2\left[\kappa_1(\bar{\phi}) - \kappa_2(\bar{\phi})\right]. \quad (32)$$

The correlation function can then be calculated in a similar fashion to Eq. (27)

$$\omega(\theta) = 4\pi^2 \int_0^\infty dx\; y^2 f^2(x) \left[y\, a\left(S_1(x)b_1(x) - S_2(x)b_2(x)\right) + 3\Omega_0\left(5(s_1 w_1(x) - s_2 w_2(x)) - 2\left(w_1(x) - w_2(x)\right)\right)\right]^2 \times \int_0^\infty dk\; k\; P(k) J_0(kx\theta). \quad (33)$$

Figure 2 shows the correlation amplitude for the same model model as in Figure 1. Here we have assumed two populations with the same selection function $S_1(x) = S_2(x)$, thus $w_1(x) = w_2(x)$. We have further assumed that $\sigma_8 = 1$ and that $b_1 = 1$, $b_2 = 1/2$. This means we have strongly and weakly correlated populations. We show A_w for $\Delta s = s_1 - s_2 = (-0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3)$. The solid curve is A_w in the absence of magnification bias $\Delta s = 0$. For a positive value of Δs the curves are always above the solid curve. At low redshift there is little magnification bias and the curves all follow the solid curve. For positive Δs the curves move significantly away from the solid line at $< z > \approx 1/2$ and then become nearly flat

out to $< z >\approx 2$. For negative Δs, the curves trace the solid line more closely, but with a significant difference at $< z >\approx 1/2$. For the curves with a substantial difference in number counts slope, the curves then hit a minimum and then rise steeply.

These curves for negative Δs may be relevant for a strongly clustered red population with small number counts slope and a weakly clustered blue population with a larger value of s.

This estimator is most useful when the intrinsic clustering is similar in the two bands, while the slopes of the number counts are very different. In the case where the intrinsic clustering is the same in the two passband, *i.e.* $\Delta\mu(m_1,\bar{\phi}) = \Delta\mu(m_2,\bar{\phi})$, there is zero contribution from the intrinsic clustering. If also the depth in comoving distance is similar, then $\kappa_1 \approx \kappa_2$ and we get that

$$\omega_{\kappa\kappa}(\theta) = 25(s_b - s_r)^2 C_{\kappa\kappa}(\theta). \tag{34}$$

This assumption is not as severe as it might seem as can be seen from the following example. Look at two source planes at distances $x_1 < x_2$ and two lightrays separated by a small angle $\bar{\theta}$. A mass fluctuation at distance x will coherently influence the convergence in the two beams only if the beams both pass through the density fluctuation. This is obviously only possible if $x < x_1$. Thus density fluctuations beyond the nearest source plane will not influence the correlation, and

$$\left\langle \kappa_1(\bar{\phi}+\bar{\theta})\kappa_2(\bar{\theta})\right\rangle \approx \left\langle \kappa_1(\bar{\phi}+\bar{\theta})\kappa_1(\bar{\theta})\right\rangle \Rightarrow C_{\kappa_1\kappa_2}(\theta) \approx C_{\kappa_1\kappa_1}(\theta). \tag{35}$$

In that case we have to a good approximation that the observed two point correlation function of the ratio of counts is given by Eq. 34, which for the ratio of very red and very blue samples gives

$$\omega_{\kappa\kappa}(\theta) \approx 3C_{\kappa\kappa}(\theta). \tag{36}$$

If at intermediate magnitudes we find that the observed correlation strengths differ by a factor C^2 we can use a different estimator of the clustering,

$$\Delta(m_1,m_2,\bar{\phi}) \equiv \left[\frac{N_{obs}(m_1,\bar{\phi})}{N_0(m_1)} - 1\right] - C\left[\frac{N_{obs}(m_2,\bar{\phi})}{N_0(m_2)} - 1\right] \tag{37}$$

$$= \Delta\mu(m_1,\bar{\phi}) - C\Delta\mu(m_2,\bar{\phi}) + 5\left[s_1\kappa_1(\bar{\phi}) - C\ s_2\kappa_2(\bar{\phi})\right] - 2\left[\kappa_1(\bar{\phi}) - C\ \kappa_2(\bar{\phi})\right]. \tag{38}$$

The correlation function can then be calculated in a similar fashion to Eq. (27)

$$\begin{aligned}\omega(\theta) &= 4\pi^2\int_0^\infty dx\ y^2 f^2(x)\left[y\ a\left(S_1(x)b_1(x) - C\ S_2(x)b_2(x)\right) + 3\Omega_0\left(5(s_1w_1(x) - C\ s_2w_2(x)) - 2\left(w_1(x) - C\ w_2(x)\right)\right)\right]^2 \\ &\quad\times\int_0^\infty dk\ k\ P(k)J_0(kx\theta).\end{aligned} \tag{39}$$

By construction, the intrinsic correlations cancel out at intermediate redshift. At higher redshifts this cancellation may not occur, but this estimator may minimize the effects of the intrinsic clustering even at faint magnitudes.

4 CORRELATION FUNCTIONS, REVISED LIMBER'S EQUATION

Suppose we have sufficiently well behaved functions $F_1(\bar{\phi})$, $F_2(\bar{\phi})$, $\delta_1(\underline{x})$, $\delta_2(\underline{x})$, $G_1(x)$, and $G_2(x)$ defined as

$$F_1(\bar{\phi}) \equiv \int_0^\infty dxG_1(x)\delta_1(\underline{x})\ ;\ F_2(\bar{\phi}) \equiv \int_0^\infty dxG_2(x)\delta_2(\underline{x})\ ;\ F(\bar{\phi}) \equiv F_1(\bar{\phi}) + F_2(\bar{\phi}). \tag{40}$$

The two-point correlation function $C_{FF}(\theta)$ of F will then be

$$\begin{aligned}C_{FF}(\theta) = 2\int_0^\infty dx \quad &\left(G_1^2(x)\int_0^\infty dq\ \xi^{11}\left(\left((x\theta)^2+q^2\right)^{1/2}\right]\right) + \\ &\left(G_2^2(x)\int_0^\infty dq\ \xi^{22}\left[\left((x\theta)^2+q^2\right)^{1/2}\right]\right) + \\ &\left(2G_1(x)G_2(x)\int_0^\infty dq\ \xi^{12}\left[\left((x\theta)^2+q^2\right)^{1/2}\right]\right),\end{aligned} \tag{41}$$

where ξ^{11}, ξ^{22}, and ξ^{12} are the two autocorrelation functions of $\delta_1(\underline{x})$ and $\delta_2(\underline{x})$ and their cross correlation function. This follows from Eq. 26 and V95a, Eqs. 37-39.

Let us take some galaxy density distribution $\delta^g(\underline{x})$ and mass density distribution $\delta^m(\underline{x})$. Then the surface density of galaxies $\Delta\mu$ and convergence κ are

$$\Delta\mu(\bar{\theta}) = \int_0^\infty dx\ y^2 S(x)\delta^g(\underline{x}), \tag{42}$$

$$\kappa(\bar{\theta}) \quad = \quad 3\Omega_0 \int_0^\infty dx\, y w(x) \delta^m(\underline{x})/a. \tag{43}$$

These equations are useful for calculating $\omega(\theta)$. We obtain

$$\begin{aligned}\omega(\theta) = 2\int_0^\infty dx \qquad & \left(y^4 S^2(x) \int_0^\infty dq\, \xi^{gg} \left[\left((x\theta)^2 + q^2 \right)^{1/2} \right] \right) + \\ & \left(9\Omega_0^2 (5s-2)^2 y^2 w^2(x) a^{-2} \int_0^\infty dq\, \xi^{mm} \left[\left((x\theta)^2 + q^2 \right)^{1/2} \right] \right) + \\ & \left(6\Omega_0 (5s-2)\, y^3 S(x) w(x) a^{-1} \int_0^\infty dq\, \xi^{gm} \left[\left((x\theta)^2 + q^2 \right)^{1/2} \right] \right).\end{aligned} \tag{44}$$

Here ξ^{gg}, ξ^{mm}, and ξ^{gm} are the galaxy-galaxy, mass-mass, and galaxy-mass correlations evaluated at the appropriate epoch. In the case where $s = 0.4$, *i.e.* no magnification bias, this reduces to the ordinary Limber's equation.

We can rephrase the correlation function calculation as an integral in redshift. This becomes simply

$$\begin{aligned}\omega(\theta) = 2\int_0^\infty dz \qquad & \left(H(z) N^2(z) \int_0^\infty dq\, \xi^{gg} \left[\left((x\theta)^2 + q^2 \right)^{1/2} \right] \right) + \\ & \left(H^{-1}(z)\, 9\Omega_0^2 (5s-2)^2 y^2 w^2(z)\, (1+z)^2 \int_0^\infty dq\, \xi^{mm} \left[\left((x\theta)^2 + q^2 \right)^{1/2} \right] \right) + \\ & \left(6\Omega_0 (5s-2)\, y\, N(z) w(z)\, (1+z) \int_0^\infty dq\, \xi^{gm} \left[\left((x\theta)^2 + q^2 \right)^{1/2} \right] \right).\end{aligned} \tag{45}$$

Here $N(z) = y^2\ S(x) H^{-1}(z)$, $w(z) = w(x)$. Notice that the normalisation integral for $N(z)$ still applies, (Eq. 19). From this equation it is possible to calculate $\omega(\theta)$ given a cosmological model *i.e.* Ω_0, Ω_Λ, and a selection function $N(z)$ if you know, or assume, the three correlation functions.

Let us make life easy and assume that the correlation functions are all power laws, not necessarily with the same slopes and evolution. For each of them assume that in proper coordinates r

$$\xi(r,z) = \left(\frac{r}{r_o} \right)^{-\gamma} (1+z)^{-(3+\epsilon)}. \tag{46}$$

Here, the exponents γ and ϵ need not be the same for the correlation functions. Even more importantly, the proper correlation lengths r_0 need not be the same. With this assumption

$$\omega(\theta) \equiv \omega^{gg}(\theta) + \omega^{mm}(\theta) + 2\,\omega^{gm}(\theta). \tag{47}$$

$$\omega^{gg}(\theta) \quad = \quad \sqrt{\pi}\, \frac{\Gamma[(\gamma-1)/2]}{\Gamma[\gamma/2]}\, r_0^\gamma\, \theta^{1-\gamma} \int_0^\infty dz\ H(z)\ N^2(z)\ x^{1-\gamma}\ (1+z)^{\gamma-3-\epsilon}, \tag{48}$$

$$\omega^{mm}(\theta) \quad = \quad \sqrt{\pi}\, \frac{\Gamma[(\gamma-1)/2]}{\Gamma[\gamma/2]}\, 9\ \Omega_0^2\ (5s-2)^2\ r_0^\gamma\ \theta^{1-\gamma} \int_0^\infty dz\ H^{-1}(z)\ w^2(z)\ y^2\ x^{1-\gamma}\ (1+z)^{\gamma-1-\epsilon}, \tag{49}$$

$$\omega^{gm}(\theta) \quad = \quad \sqrt{\pi}\, \frac{\Gamma[(\gamma-1)/2]}{\Gamma[\gamma/2]}\, 3\ \Omega_0\ (5s-2)\ r_0^\gamma\ \theta^{1-\gamma} \int_0^\infty dz\ N(z)\ w(z)\ y\ x^{1-\gamma}\ (1+z)^{\gamma-2-\epsilon}. \tag{50}$$

The term for the intrinsic clustering ω^{gg} reduces to the same result as (BSM), Eqs. (6,7) when taking into account differences in notation. Let us make life even easier for ourselves and assume that γ is the same for all three correlation functions. However allow ϵ to be different so that $\epsilon^{mm} = \epsilon + \Delta\epsilon$, and $\epsilon^{gm} = \epsilon + \Delta\epsilon/2$. Further assume a linear bias model so that today $\xi^{gg}(z=0) \equiv b^2\ \xi^{mm}(z=0)$, and $\xi^{gm}(z=0) \equiv b\ \xi^{gm}(z=0)$. Now r_0 is the correlation length of the galaxies. This simplifies the integral for $\omega(\theta)$.

$$\begin{aligned}\omega(\theta) \quad &= \quad \sqrt{\pi}\, \frac{\Gamma[(\gamma-1)/2]}{\Gamma[\gamma/2]}\, r_0^\gamma\ \theta^{1-\gamma} \\ &\times \quad \int_0^\infty dz\ H(z)\ \left[N(z) +\ 3\ \frac{\Omega_0}{b}\ (5s-2)\ w(z)\ y\ (1+z)^{1-\Delta\epsilon/2}\ H^{-1}(z) \right]^2 x^{1-\gamma}\ (1+z)^{\gamma-3-\epsilon}.\end{aligned} \tag{51}$$

Parametrized this way it is possible to do an analysis of $\omega(\theta)$ in the same way as BSM while including the effects of the magnification bias. For a given magnitude limit, assume a cosmological model, *i.e.* Ω_0, Ω_Λ. Then choose γ, ϵ, $\Delta\epsilon$, b, and r_0. For a given $N(z)$ this will then predict the observed $\omega(\theta)$.

As stated in BSM, for $\gamma = 1.8$, linear theory predicts that $\epsilon = 0.8$, clustering fixed in comoving coordinates will give $\epsilon = -1.2$, while clustering fixed in proper coordinates will have $\epsilon = 0$.

If we have further information about κ we can improve on our measurements. If we can measure image shapes and position angles, then we can infer the gravitational shear field p. We can then measure the two point correlation function $C_{pp}(\theta)$. In

the weak limit, $C_{\kappa\kappa}(\theta) = C_{pp}(\theta)$ and $\omega^{mm}(\theta) = (5s-2)^2 C_{\kappa\kappa}(\theta)$. Thus we can remove ω^{mm} from our measurements. This leaves us with the true clustering of the galaxies plus the cross term galaxy-mass. If the observational data is of sufficiently high quality, we can estimate p at any given point on the sky. This estimate is of course smoothed over a finite area. If we can measure p across the sky, we can infer κ through an inversion procedure such as demonstrated by Kaiser & Squires (1993), or Seitz & Schneider (1995). With these methods we can obtain an unbiased estimate of κ. There is no problem with non-linearities since we are certainly in the weak limit and there is no ambiguity with the mean surface density since it is by assumption zero. If we know κ then we simply subtract the term $(5s-2)\kappa(\bar{\phi}) \times N_0(m)$ in Eq. 2. We thus retain an unbiased estimate of the true galaxy clustering.

With this measurement we can thus separate the intrinsic clustering of galaxies, the clustering of the mass, and the galaxy-mass correlation. In other words we can separate the galaxy clustering evolution from the mass clustering evolution, and we can also measure how well light traces mass.

5 DISCUSSION

The standard way of measuring the galaxy number density fluctuations at intermediate redshifts $z \lesssim 1$ is through the angular two-point correlation function $\omega(\theta)$. Measuring $\omega(\theta)$ is in principle straightforward. You measure the positions and magnitudes of the galaxies and measure the number of galaxy pairs relative to random. Small distortions in the telescope optics do not influence the results and it is not necessary to measure the shape and orientation of the galaxies. The interpretation of $\omega(\theta)$ in terms of the three-dimensional correlation function $\xi(r)$ is complicated through largely unknown luminosity and density evolution. Even if $\xi(r)$ were known accurately, this would only tell us the galaxy distribution, not necessarily the mass distribution. The mass distribution would then have to be inferred through a model dependent biasing scheme, or some other modeling scheme.

Weak lensing by large scale structure is a direct measure of the mass distribution and it circumvents inferring the mass distribution from the galaxy distribution. The problem with weak lensing is that it is weak. It requires measuring the shape and orientation of faint galaxy images. This is observationally feasible, but quite difficult, since the images are small and faint, and the possible systematic errors are the limiting factors. However, the payoff is immense in terms of measuring cosmological parameters and the power spectrum of density fluctuations.

We have presented a new way of measuring the mass density fluctuations at intermediate redshifts by measuring the angular two-point correlation function $\omega(\theta)$. At sufficiently faint magnitudes, the observed $\omega(\theta)$ will be dominated by weak lensing. The method combines the relative ease at which $\omega(\theta)$ can be determined, with the relatively simple theoretical interpretation of weak lensing in terms of cosmological parameters and the statistical properties of the mass distribution.

At relatively bright magnitudes, $R \lesssim 23$, weak lensing is expected to be unimportant. If the slope of the number counts is greater/less than 0.4, the amplitude of $\omega(\theta)$ will decrease slower/faster than expected from Limber's equation. In both cases the amplitude will hit a minimum and then increase with limiting magnitude.

An equivalent measure is the ratio of number counts in two different passbands as a function of position in the sky. This measure can, by carefully choosing the passbands and magnitudes, be tuned to be nearly independent of the true clustering of galaxies. In that case, $\omega(\theta)$ is a straight measure of weak lensing and the theoretical interpretation is considerably simpler.

In summary: The angular two-point correlation function of galaxies is affected by weak lensing by large scale structure through the magnification bias. At faint magnitudes this can be a signicant effect and must be included in calculations. For a properly designed experiment, $\omega(\theta)$ can be used to infer the clustering of the mass, the clustering of the galaxies, and how well light traces mass.

More work needs to be done.

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